

# Package ‘HMMcopula’

July 21, 2025

**Type** Package

**Title** Markov Regime Switching Copula Models Estimation and Goodness-of-Fit

**Version** 1.1.0

**Maintainer** Bruno N Remillard <bruno.remillard@hec.ca>

**Description** Estimation procedures and goodness-of-fit test for several Markov regime switching models and mixtures of bivariate copula models. The goodness-of-fit test is based on a Cramer-von Mises statistic and uses Rosenblatt’s transform and parametric bootstrap to estimate the p-value. The proposed methodologies are described in Nasri, Remillard and Thioub (2020) <[doi:10.1002/cjs.11534](https://doi.org/10.1002/cjs.11534)>.

**License** GPL (>= 2)

**Encoding** UTF-8

**Depends** mvtnorm, foreach, doParallel, copula

**RoxygenNote** 7.3.2

**NeedsCompilation** no

**Author** Bouchra R. Nasri [aut],  
Bruno N Remillard [aut, cre, cph],  
Mamadou Yamar Thioub [aut],  
Romanic Pieugueu [aut]

**Repository** CRAN

**Date/Publication** 2024-10-02 16:40:09 UTC

## Contents

CopulaFamiliesCDF . . . . .	2
dilog . . . . .	3
EstHMMCop . . . . .	3
EstKendallTau . . . . .	4
EstMixtureCop . . . . .	5
GofHMMCop . . . . .	6
GofMixtureCop . . . . .	7
KendallTau . . . . .	8

ParamCop . . . . .	8
ParamTau . . . . .	9
RosenblattClayton . . . . .	9
RosenblattFrank . . . . .	10
RosenblattGaussian . . . . .	10
RosenblattGumbel . . . . .	11
RosenblattStudent . . . . .	11
SimHMMCop . . . . .	12
SimMarkovChain . . . . .	12
SimMixtureCop . . . . .	13
SnB . . . . .	14
Tau2Rho . . . . .	14
<b>Index</b>	<b>15</b>

---

CopulaFamiliesCDF	<i>CopulaFamiliesCDF</i>
-------------------	--------------------------

---

## Description

COPULACDF Cumulative probability function for a copula with linear correlation parameters RHO

## Usage

```
CopulaFamiliesCDF(family, u, ...)
```

## Arguments

family	copula family= "gaussian" , "t" , "clayton" , "frank" , "gumbel"
u	is an N-by-P matrix of values in [0,1], representing N points in the P-dimensional unit hypercube
...	additionnal parameter like RHO a P-by-P correlation matrix.

## Value

$Y = \text{COPULACDF}(\text{'Gaussian'}, U, \text{RHO})$  returns the cumulative probability of the Gaussian copula with linear correlation parameters RHO, evaluated at the points in U. U is an N-by-P matrix of values in [0,1], representing N points in the P-dimensional unit hypercube. RHO is a P-by-P correlation matrix. If U is an N-by-2 matrix, RHO may also be a scalar correlation coefficient.

$Y = \text{COPULACDF}(\text{'t'}, U, \text{RHO}, \text{NU})$  returns the cumulative probability of the t copula with linear correlation parameters RHO and degrees of freedom parameter NU, evaluated at the points in U. U is an N-by-P matrix of values in [0,1]. RHO is a P-by-P correlation matrix. If U is an N-by-2 matrix, RHO may also be a scalar correlation coefficient.

$Y = \text{COPULACDF}(\text{FAMILY}, U, \text{ALPHA})$  returns the cumulative probability of the bivariate Archimedean copula determined by FAMILY, with scalar parameter ALPHA, evaluated at the points in U. FAMILY is 'Clayton', 'Frank', or 'Gumbel'. U is an N-by-2 matrix of values in [0,1].

**Examples**

```
u = seq(0,1,0.1);
U1=matrix(rep(u,length(u)),nrow=length(u),byrow = TRUE); U2=t(U1)
F = CopulaFamiliesCDF('clayton',cbind(c(U1), c(U2)),1)
```

---

dilog	<i>Dilogarithm function</i>
-------	-----------------------------

---

**Description**

Computation of the dilogarithm function by numerical integration.

**Usage**

```
dilog(x)
```

**Arguments**

x	a real number
---	---------------

**Value**

out	dilogarithm
-----	-------------

---

EstHMMCop	<i>Estimation of bivariate Markov regime switching bivariate copula model</i>
-----------	---

---

**Description**

Estimation of parameters from a bivariate Markov regime switching bivariate copula model

**Usage**

```
EstHMMCop(y, reg, family, max_iter, eps)
```

**Arguments**

y	(nx2) data matrix (observations or residuals) that will be transformed to pseudo-observations
reg	number of regimes
family	'gaussian', 't', 'clayton', 'frank', 'gumbel'
max_iter	maximum number of iterations of the EM algorithm
eps	precision (stopping criteria); suggestion 0.0001.

**Value**

theta	(1 x reg) estimated parameter of the copula according to CRAN copula package (except for Frank copula, where theta = log(theta_R_Package)) for each regime (except for degrees of freedom)
dof	estimated degree of freedom, only for the Student copula
Q	(reg x reg) estimated transition matrix
eta	(n x reg) conditional probabilities of being in regime k at time t given observations up to time t
tau	estimated Kendall tau for each regime
U	(n x 2) matrix of Rosenblatt transforms
cvm	Cramer-von-Mises statistic for goodness-of-fit
W	regime probabilities for the conditional distribution given the past Kendall's tau

**Author(s)**

Mamadou Yamar Thioub and Bruno Remillard, April 12, 2018

**References**

<doi::10.1002/cjs.11534>

**Examples**

```
Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2) ; kendallTau <- c(0.3 ,0.7) ;
data <- SimHMMCop(Q, 'clayton', kendallTau, 10)$SimData;
estimations <- EstHMMCop(data,2,'clayton',10000,0.0001)
```

---

EstKendallTau

*Sample Kendall's tau Estimation*

---

**Description**

This function estimates the sample Kendall's tau of a bivariate data matrix

**Usage**

```
EstKendallTau(X)
```

**Arguments**

X (n x 2) matrix

**Value**

KendallTau estimated sample Kendall's tau of the data

---

EstMixtureCop	<i>Estimation of bivariate mixture bivariate copula model</i>
---------------	---

---

**Description**

Estimation of parameters from a mixture of bivariate copula models

**Usage**

```
EstMixtureCop(y, reg, family, max_iter, eps)
```

**Arguments**

<code>y</code>	( <code>nx2</code> ) data matrix (observations or residuals) that will be transformed to pseudo-observations
<code>reg</code>	number of regimes
<code>family</code>	'gaussian', 't', 'clayton', 'frank', 'gumbel'
<code>max_iter</code>	maximum number of iterations of the EM algorithm
<code>eps</code>	precision (stopping criteria); suggestion 0.0001.

**Value**

<code>theta</code>	( <code>1 x reg</code> ) estimated parameter of the copula according to CRAN copula package (except for Frank copula, where <code>theta = log(theta_R_Package)</code> ) for each component (except for degrees of freedom)
<code>dof</code>	estimated degree of freedom, only for the Student copula
<code>Q</code>	( <code>1 x reg</code> ) estimated weights vector
<code>eta</code>	( <code>n x reg</code> ) conditional probabilities of being in regime <code>k</code> at time <code>t</code> given observations up to time <code>t</code>
<code>tau</code>	estimated Kendall tau for each regime
<code>U</code>	( <code>n x 2</code> ) matrix of Rosenblatt transforms
<code>cvm</code>	Cramer-von-Mises statistic for goodness-of-fit

**Author(s)**

Mamadou Yamar Thioub and Bruno Remillard, April 12, 2018

**References**

<doi::10.1002/cjs.11534>

GofHMMCop

*Goodness-of-fit of Markov regime switching bivariate copula model***Description**

Goodness-of-fit test of a Markov regime switching bivariate copula model

**Usage**

```
GofHMMCop(R, reg, family, max_iter, eps, n_sample, n_cores)
```

**Arguments**

R	(n x 2) data matrix that will be transformed to pseudo-observations
reg	number of regimes
family	'gaussian', 't', 'clayton', 'frank', 'gumbel'
max_iter	maximum number of iterations of the EM algorithm
eps	precision (stopping criteria); suggestion 0.0001
n_sample	number of bootstrap; suggestion 1000
n_cores	number of cores to use in the parallel computing

**Value**

pvalue	pvalue (significant when the result is greater than 5)
theta	(1 x reg) estimated parameter of the copula according to CRAN copula package (except for Frank copula, where theta = log(theta_R_Package)) for each regime (except for degrees of freedom)
dof	estimated degree of freedom, only for the Student copula
Q	(reg x reg) estimated transition matrix
eta	(n x reg) conditional probabilities of being in regime k at time t given observations up to time t
tau	estimated Kendall tau for each regime
U	(n x 2) matrix of Rosenblatt transforms
cvm	Cramer-von-Mises statistic for goodness-of-fit
W	regime probabilities for the conditional distribution given the past Kendall's tau

**References**

<doi::10.1002/cjs.11534>

---

GofMixtureCop                      *Goodness-of-fit of mixture bivariate copula model*

---

**Description**

Goodness-of-fit test of a mixture bivariate copula model

**Usage**

GofMixtureCop(R, reg, family, max\_iter, eps, n\_sample, n\_cores)

**Arguments**

R	(nx2) data matrix (observations or residuals) that will be transformed to pseudo-observations
reg	number of regimes
family	'gaussian' , 't' , 'clayton' , 'frank' , 'gumbel'
max_iter	maximum number of iterations of the EM algorithm
eps	precision (stopping criteria); suggestion 0.0001
n_sample	number of bootstrap; suggestion 1000
n_cores	number of cores to use in the parallel computing

**Value**

pvalue	pvalue (significant when the result is greater than 5)
theta	(1 x reg) estimated parameter of the copula according to CRAN copula package (except for Frank copula, where theta = log(theta_R_Package)) for each component (except for degrees of freedom)
dof	estimated degree of freedom, only for the Student copula
Q	(1 x reg) estimated weights vector
eta	(n x reg) conditional probabilities of being in regime k at time t given observations up to time t
tau	estimated Kendall tau for each regime
U	(n x 2) matrix of Rosenblatt transforms
cvm	Cramer-von-Mises statistic for goodness-of-fit

**Author(s)**

By Bruno Remillard, Nov 28, 2010

**References**

<doi::10.1002/cjs.11534>

---

KendallTau	<i>Kendall's tau of a copula</i>
------------	----------------------------------

---

**Description**

Computation of Kendall's tau of a copula family with an unconstrained parameter alpha.

**Usage**

```
KendallTau(family, alpha)
```

**Arguments**

family	"gaussian" , "t" , "clayton" , "frank" , "gumbel"
alpha	unconstrained parameters of the copula family

**Value**

tau	estimated Kendall's tau
-----	-------------------------

---

ParamCop	<i>Theta estimation</i>
----------	-------------------------

---

**Description**

Parameters of a copula according to CRAN copula package (except for Frank copula, where theta = log(theta\_R\_Package)), corresponding to the unconstrained parameters alpha.

**Usage**

```
ParamCop(family, alpha)
```

**Arguments**

family	"gaussian" , "t" , "clayton" , "frank" , "gumbel"
alpha	unconstrained parameters of the copula family

**Value**

theta	matlab parameters
-------	-------------------



---

ParamTau	<i>Alpha estimation</i>
----------	-------------------------

---

**Description**

Unconstrained parameter for a given Kendall's tau.

**Usage**

```
ParamTau(family, tau)
```

**Arguments**

family	'gaussian', 't', 'clayton', 'frank', 'gumbel'
tau	Kendall's tau of the copula family

**Value**

alpha	estimated unconstrained parameter
-------	-----------------------------------

---

RosenblattClayton	<i>Rosenblatt transform for Clayton copula</i>
-------------------	--

---

**Description**

Computation of the Rosenblatt transform for Clayton's copula

**Usage**

```
RosenblattClayton(u, theta)
```

**Arguments**

u	(n x d) matrix of pseudos-observations (normalized ranks)
theta	parameter of the Clayton copula

**Value**

R	Rosenblatt transform
---	----------------------

---

RosenblattFrank	<i>Rosenblatt transform for Frank copula</i>
-----------------	--

---

**Description**

Computation of the Rosenblatt transform for Frank's copula

**Usage**

RosenblattFrank(U, theta)

**Arguments**

U	(n x d) matrix of pseudos-observations (normalized ranks)
theta	parameter of the Frank copula

**Value**

R	Rosenblatt transform
---	----------------------

---

RosenblattGaussian	<i>Rosenblatt transform for Gaussian copula</i>
--------------------	---

---

**Description**

Computation of the Rosenblatt transform for the Gaussian copula

**Usage**

RosenblattGaussian(u, rho)

**Arguments**

u	(n x d) matrix of pseudos-observations (normalized ranks)
rho	(d x d) correlation matrix, or the correlation coefficient (if, d = 2)

**Value**

R	Rosenblatt transform
---	----------------------

---

RosenblattGumbel	<i>Rosenblatt transform for Gumbel copula</i>
------------------	---

---

**Description**

Computation of the Rosenblatt transform for Gumbel's copula

**Usage**

```
RosenblattGumbel(U, theta)
```

**Arguments**

U	(n x d) matrix of pseudos-observations (normalized ranks)
theta	parameter of the Gumbel copula

**Value**

R	Rosenblatt transform
---	----------------------

---

RosenblattStudent	<i>Rosenblatt transform for Student copula</i>
-------------------	--

---

**Description**

Computation of the Rosenblatt transform for the Student copula

**Usage**

```
RosenblattStudent(u, rho, nu)
```

**Arguments**

u	(n x d) matrix of pseudos-observations (normalized ranks)
rho	(d x d) correlation matrix
nu	degrees of freedom

**Value**

R	Rosenblatt transform
---	----------------------

---

 SimHMMCop

*Simulation of bivariate Markov regime switching copula model*


---

**Description**

Simulation of values from a bivariate Markov regime switching copula model

**Usage**

```
SimHMMCop(Q, family, KendallTau, n, DoF)
```

**Arguments**

Q	Transition probability matrix (d x d);
family	'gaussian', 't', 'clayton', 'frank', 'gumbel'
KendallTau	Kendall's rank correlation
n	number of simulated vectors
DoF	degree of freedom only for the Student copula

**Value**

SimData	Simulated Data
MC	Markov chain regimes
alpha	parameters alpha

**Examples**

```
Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2) ; kendallTau <- c(0.3 ,0.7) ;
simulations <- SimHMMCop(Q, 'gumbel', kendallTau, 300)
```

---

 SimMarkovChain

*Markov chain simulation*


---

**Description**

Simulation of n consecutive values of a Markov chain with transition matrix Q, starting from a state eta0 or the uniform distribution on the set 1,..., r.

**Usage**

```
SimMarkovChain(Q, n, eta0)
```

**Arguments**

Q	Transition probability matrix (d x d)
n	number of simulated vectors
eta0	variable eta

**Value**

x	Simulated Markov chain sequence
---	---------------------------------

---

SimMixtureCop	<i>Simulation of bivariate mixture copula model</i>
---------------	---

---

**Description**

Simulation of observations from a bivariate mixture copula model

**Usage**

```
SimMixtureCop(Q, family, KendallTau, n, DoF)
```

**Arguments**

Q	Weights vector (1 x component);
family	'gaussian', 't', 'clayton', 'frank', 'gumbel'
KendallTau	Kendall's rank correlation
n	number of simulated vectors
DoF	vector of degree of freedom only for the Student copula

**Value**

SimData	Simulated Data
MC	Markov chain regimes
alpha	parameters alpha

**Examples**

```
Q <- matrix(c(0.8, 0.2),1,2) ; kendallTau <- c(0.3 ,0.7) ;
simulations <- SimMixtureCop(Q, 'gaussian', kendallTau, 300)
```

---

SnB	<i>Cramer-von Mises statistic SnB for GOF based on the Rosenblatt transform</i>
-----	---

---

**Description**

Computation of the Cramer-von Mises statistic SnB for GOF based on the Rosenblatt transform

**Usage**

SnB(E)

**Arguments**

E (n x d) matrix of pseudos-observations (normalized ranks)

**Value**

Sn Cramer-von Mises statistic

---

Tau2Rho	<i>Spearman's rho</i>
---------	-----------------------

---

**Description**

Value of Spearman's rho corresponding to a constrained (matlab) parameter theta for a copula family.

**Usage**

Tau2Rho(family, theta)

**Arguments**

family 'gaussian', 't', 'clayton', 'frank', 'gumbel'

theta parameter of the copula according to CRAN copula package (except for Frank copula, where theta = log(theta\_R\_Package))

**Value**

rho estimated Spearman's rho

# Index

CopulaFamiliesCDF, [2](#)

dilog, [3](#)

EstHMMCop, [3](#)

EstKendallTau, [4](#)

EstMixtureCop, [5](#)

GofHMMCop, [6](#)

GofMixtureCop, [7](#)

KendallTau, [8](#)

ParamCop, [8](#)

ParamTau, [9](#)

RosenblattClayton, [9](#)

RosenblattFrank, [10](#)

RosenblattGaussian, [10](#)

RosenblattGumbel, [11](#)

RosenblattStudent, [11](#)

SimHMMCop, [12](#)

SimMarkovChain, [12](#)

SimMixtureCop, [13](#)

SnB, [14](#)

Tau2Rho, [14](#)